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Impurity scattering in metallic carbon nanotubes with superconducting pair potentials

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Abstract. The effects of the superconducting pair potential on the impurity scattering processes in metallic carbon nanotubes are studied theoretically. The backward scattering of electrons vanishes in the normal state. In the presence of superconducting pair correlations, the backward scatterings of electron- and hole-like quasiparticles vanish, too. The impurity gives rise to backward scatterings of holes for incident electrons, and it also induces backward scatterings of electrons for incident holes. Negative and positive currents induced by such scatterings between electrons and holes cancel each other. Therefore, the nonmagnetic impurity does not hinder the supercurrent in the regions where the superconducting proximity effects occur. Comparisons with experiment are discussed.

1. Introduction

Recent investigations [1, 2] show that the superconducting proximity effect occurs when carbon nanotubes contact with conventional superconducting metals and wires. The superconducting energy gap appears in the tunnelling density of states below the critical temperature T_c. On the other hand, recent theories discuss the nature of the exceptionally ballistic conduction [3] and the absence of backward scattering [4] in metallic carbon nanotubes with impurity potentials in the normal states (not in the superconducting states). Such peculiar properties might be related to the experimental realization of nanostructures with quantum electronic conductions [5, 6]. However, impurity scattering properties in the presence of superconducting pair potentials have not been investigated theoretically so much. Therefore, there is now an urgent need to study how the peculiar scattering properties in the normal states will change when the superconducting proximity effects occur in metallic carbon nanotubes.

In this paper, we study the effects of the superconducting pair potential on the impurity scattering processes in metallic carbon nanotubes. We use the continuum $\boldsymbol{k} \cdot \boldsymbol{p}$ model for the electronic states in order to consider scattering processes in the normal state and also in the state with the superconducting pair potential. We find that the scattering matrix is diagonal and the off-diagonal matrix elements vanish in the normal state. Such absence of

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backward scattering has been discussed recently [4], too. Next, we consider the effects of the superconducting pair correlations. We find the absence of backward scatterings of electron- and hole-like quasiparticles in the presence of superconducting proximity effects. The off-diagonal 2×2 submatrix has diagonal matrix elements whose magnitudes are proportional to Δ. Negative and positive currents induced by such scatterings between electrons and holes cancel each other. Therefore, the nonmagnetic impurity *does not hinder the supercurrent* in the regions where the superconducting proximity effects occur. This finding is interesting in view of the recent experimental progress as regards the superconducting proximity effects of carbon nanotubes [1, 2].

In the next section, we explain our model and introduce propagators of the normal state and the Nambu representation. In section 3, we consider impurity scattering in the normal state. In section 4, we discuss the effects of superconducting pair correlations. Section 5 is devoted to the discussion of possible complex pairings. A summary is given in section 6.

2. Model

We will study metallic carbon nanotubes with a superconducting pair potential. The model is as follows:

$$H = H_{\rm tube} + H_{\rm pair}. \tag{1}$$

$H_{\rm tube}$ describes the electronic states of the carbon nanotubes, and the model based on the $\boldsymbol{k} \cdot \boldsymbol{p}$ approximation [4, 7] represents electronic systems in the continuum medium. The second term, $H_{\rm pair}$, is the pair potential term present owing to the proximity effect.

The Hamiltonian in the $\boldsymbol{k} \cdot \boldsymbol{p}$ approximation [4, 7] in the second-quantized representation has the following form:

$$H_{\rm tube} = \sum_{\boldsymbol{k},\sigma} \Psi^\dagger_{\boldsymbol{k},\sigma} E_{\boldsymbol{k}} \Psi_{\boldsymbol{k},\sigma} \tag{2}$$

where $E_{\boldsymbol{k}}$ is an energy matrix:

$$E_{\boldsymbol{k}} = \begin{pmatrix} 0 & \gamma(k_x - {\rm i}k_y) & 0 & 0 \\ \gamma(k_x + {\rm i}k_y) & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma(k_x + {\rm i}k_y) \\ 0 & 0 & \gamma(k_x - {\rm i}k_y) & 0 \end{pmatrix}. \tag{3}$$

$\boldsymbol{k} = (k_x, k_y)$ and $\Psi_{\boldsymbol{k},\sigma}$ is an annihilation operator with four components:

$$\Psi^\dagger_{\boldsymbol{k},\sigma} = (\psi^{(1)\dagger}_{\boldsymbol{k},\sigma}, \psi^{(2)\dagger}_{\boldsymbol{k},\sigma}, \psi^{(3)\dagger}_{\boldsymbol{k},\sigma}, \psi^{(4)\dagger}_{\boldsymbol{k},\sigma}).$$

Here, the fist and second elements indicate an electron at the A- and B-sublattice points around the Fermi point K of the graphite, respectively. The third and fourth elements indicate an electron at the A- and B-sublattice points around the Fermi point K′. The quantity γ is defined as $\gamma \equiv (\sqrt{3}/2)a\gamma_0$, where a is the bond length of the graphite plane and γ_0 ($\simeq$2.7 eV) is the resonance integral connecting neighbouring carbon atoms. When the above matrix is diagonalized, we obtain the dispersion relation

$$E_\pm = \pm\gamma\sqrt{\kappa^2_{\nu\phi}(n) + k_y^2}$$

where k_y is parallel with the axis of the nanotube, $\kappa_{\nu\phi}(n) = (2\pi/L)(n + \phi - \nu/3)$, L is the circumference of the nanotube, n (=0, $\pm$1, $\pm$2, $\ldots$) is the index for the bands, ϕ is the magnetic flux in units of the flux quantum, and ν (=0, 1, or 2) specifies the boundary condition in the x-direction. The metallic and semiconducting nanotubes are characterized by $\nu = 0$

and $\nu = 1$ (or 2), respectively. Hereafter, we consider the case where $\phi = 0$ and the metallic nanotubes with $\nu = 0$.

The second term in equation (1) is the pair potential

$$H_{\rm pair} = \Delta \sum_{\boldsymbol{k}} (\psi^{(1)\dagger}_{\boldsymbol{k},\uparrow} \psi^{(1)\dagger}_{-\boldsymbol{k},\downarrow} + \psi^{(2)\dagger}_{\boldsymbol{k},\uparrow} \psi^{(2)\dagger}_{-\boldsymbol{k},\downarrow} + \psi^{(3)\dagger}_{\boldsymbol{k},\uparrow} \psi^{(3)\dagger}_{-\boldsymbol{k},\downarrow} + \psi^{(4)\dagger}_{\boldsymbol{k},\uparrow} \psi^{(4)\dagger}_{-\boldsymbol{k},\downarrow} + \text{h.c.}) \tag{4}$$

where Δ is the strength of the superconducting pair correlation. In principle, Δ can have spatial dependence. However, we assume a constant Δ for simplicity. This corresponds to the case where the spatial extent of the regions where the proximity effect occurs is as long as the superconducting coherence length. After transforming to a representation where the $E = \pm\gamma|\boldsymbol{k}|$ branches crossing the Fermi energy are diagonal:

$$\begin{pmatrix} \phi^{(1)}_{\boldsymbol{k},\uparrow} \\ \phi^{(2)}_{\boldsymbol{k},\uparrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\mathrm{i}\,\mathrm{sgn}\,k_y \\ 1 & \mathrm{i}\,\mathrm{sgn}\,k_y \end{pmatrix} \begin{pmatrix} \psi^{(1)}_{\boldsymbol{k},\uparrow} \\ \psi^{(2)}_{\boldsymbol{k},\uparrow} \end{pmatrix} \tag{5}$$

and

$$\begin{pmatrix} \phi^{(3)}_{\boldsymbol{k},\uparrow} \\ \phi^{(4)}_{\boldsymbol{k},\uparrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \mathrm{i}\,\mathrm{sgn}\,k_y \\ 1 & -\mathrm{i}\,\mathrm{sgn}\,k_y \end{pmatrix} \begin{pmatrix} \psi^{(3)}_{\boldsymbol{k},\uparrow} \\ \psi^{(4)}_{\boldsymbol{k},\uparrow} \end{pmatrix} \tag{6}$$

we obtain the following relation:

$$H_{\rm pair} = \Delta \sum_{\boldsymbol{k}} (\phi^{(1)\dagger}_{\boldsymbol{k},\uparrow} \phi^{(1)\dagger}_{-\boldsymbol{k},\downarrow} + \phi^{(2)\dagger}_{\boldsymbol{k},\uparrow} \phi^{(2)\dagger}_{-\boldsymbol{k},\downarrow} + \phi^{(3)\dagger}_{\boldsymbol{k},\uparrow} \phi^{(3)\dagger}_{-\boldsymbol{k},\downarrow} + \phi^{(4)\dagger}_{\boldsymbol{k},\uparrow} \phi^{(4)\dagger}_{-\boldsymbol{k},\downarrow} + \text{h.c.}). \tag{7}$$

This indicates that the superconducting pair correlation Δ is a correlation between the electron with the wavenumber $\boldsymbol{k}$ and spin $\uparrow$ and the electron with the wavenumber $-\boldsymbol{k}$ and spin $\downarrow$. In other words, the correlation Δ is a pairing of an s-wave state.

The propagator of the electrons on the nanotube is defined in matrix form:

$$G(\boldsymbol{k}, \tau) = -\langle T_\tau \Psi_{\boldsymbol{k},\sigma}(\tau) \Psi^\dagger_{\boldsymbol{k},\sigma}(0)\rangle \tag{8}$$

where T_τ is the time ordering operator with respect to the imaginary time τ and $\Psi_{\boldsymbol{k},\sigma}(\tau) = \exp(H\tau)\Psi_{\boldsymbol{k},\sigma}\exp(-H\tau)$. The Fourier transform of G is calculated as

$$G^{-1}(\boldsymbol{k}, \mathrm{i}\omega_n) = \begin{pmatrix} G_{\rm K}^{-1} & 0 \\ 0 & G_{\rm K'}^{-1} \end{pmatrix} \tag{9}$$

where $\omega_n = (2n+1)\pi T$ is the odd Matsubara frequency for fermions. The components of G are written explicitly as follows:

$$G_{\rm K}^{-1}(\boldsymbol{k}, \mathrm{i}\omega_n) = \begin{pmatrix} \mathrm{i}\omega_n & -\gamma(k_x - \mathrm{i}k_y) \\ -\gamma(k_x + \mathrm{i}k_y) & \mathrm{i}\omega_n \end{pmatrix} \tag{10}$$

and

$$G_{\rm K'}^{-1}(\boldsymbol{k}, \mathrm{i}\omega_n) = \begin{pmatrix} \mathrm{i}\omega_n & -\gamma(k_x + \mathrm{i}k_y) \\ -\gamma(k_x - \mathrm{i}k_y) & \mathrm{i}\omega_n \end{pmatrix}. \tag{11}$$

In order to describe the superconducting pair correlations, it is useful to introduce the Nambu representation:

$$\tilde{\Psi}^\dagger_{\rm K}(\boldsymbol{k}) = (\psi^{(1)\dagger}_{\boldsymbol{k},\uparrow}, \psi^{(2)\dagger}_{\boldsymbol{k},\uparrow}, \psi^{(1)}_{-\boldsymbol{k},\downarrow}, \psi^{(2)}_{-\boldsymbol{k},\downarrow}) \qquad \tilde{\Psi}^\dagger_{\rm K'}(\boldsymbol{k}) = (\psi^{(3)\dagger}_{\boldsymbol{k},\uparrow}, \psi^{(4)\dagger}_{\boldsymbol{k},\uparrow}, \psi^{(3)}_{-\boldsymbol{k},\downarrow}, \psi^{(4)}_{-\boldsymbol{k},\downarrow}).$$

The propagator with the pair correlation is defined in matrix form:

$$\tilde{G}_{\rm K}(\boldsymbol{k}, \tau) = -\langle T_\tau \tilde{\Psi}_{\rm K}(\boldsymbol{k}, \tau) \tilde{\Psi}^\dagger_{\rm K}(\boldsymbol{k}, 0)\rangle \tag{12}$$

and

$$\tilde{G}_{\rm K'}(\boldsymbol{k}, \tau) = -\langle T_\tau \tilde{\Psi}_{\rm K'}(\boldsymbol{k}, \tau) \tilde{\Psi}^\dagger_{\rm K'}(\boldsymbol{k}, 0)\rangle. \tag{13}$$

Their Fourier transforms are calculated as

$$\tilde{G}_{\mathrm{K}}^{-1}(\boldsymbol{k}, \mathrm{i}\omega_n) = \begin{pmatrix} \mathrm{i}\omega_n & -\gamma(k_x - \mathrm{i}k_y) & -\Delta & 0 \\ -\gamma(k_x + \mathrm{i}k_y) & \mathrm{i}\omega_n & 0 & -\Delta \\ -\Delta & 0 & \mathrm{i}\omega_n & -\gamma(-k_x + \mathrm{i}k_y) \\ 0 & -\Delta & -\gamma(-k_x - \mathrm{i}k_y) & \mathrm{i}\omega_n \end{pmatrix} \quad (14)$$

and

$$\tilde{G}_{\mathrm{K}'}^{-1}(\boldsymbol{k}, \mathrm{i}\omega_n) = \begin{pmatrix} \mathrm{i}\omega_n & -\gamma(k_x + \mathrm{i}k_y) & -\Delta & 0 \\ -\gamma(k_x - \mathrm{i}k_y) & \mathrm{i}\omega_n & 0 & -\Delta \\ -\Delta & 0 & \mathrm{i}\omega_n & -\gamma(-k_x - \mathrm{i}k_y) \\ 0 & -\Delta & -\gamma(-k_x + \mathrm{i}k_y) & \mathrm{i}\omega_n \end{pmatrix}. \quad (15)$$

The dispersion relation of the quasiparticles becomes

$$E = \pm\sqrt{\gamma^2(k_x^2 + k_y^2) + \Delta^2}.$$

The dispersions with plus and minus signs are twofold degenerate.

We note that there are several characteristic parameters of metallic carbon nanotubes. The total carbon number N_s is given by $N_s = A \times L \div (\sqrt{3}a^2/2) \times 2 = 4AL/\sqrt{3}a^2$, where A is the length of the nanotube and $\sqrt{3}a^2/2$ is the area of the unit cell. There are two carbons in one unit cell, so the factor 2 is present. The density of states near the Fermi energy $E = 0$ is constant, and it is calculated as

$$\rho(E) = (A/2\pi) \int_{-\infty}^{\infty} \mathrm{d}k_y \, \delta(E - \gamma k_y) = aN_s/4\pi L\gamma_0.$$

Because two sites in the discrete model correspond to one site in the continuum $\boldsymbol{k} \cdot \boldsymbol{p}$ model, the density of sites in the continuum model is given by $\rho \equiv \rho(E)|_{E=0} = a/2\pi L\gamma_0$.

3. Impurity scattering in normal nanotubes

Now, we consider the impurity scattering in the normal-metallic nanotubes. We take into account the single-impurity potential located at the point $\boldsymbol{r}_0$:

$$H_{\mathrm{imp}} = I \sum_{\boldsymbol{k},\boldsymbol{p},\sigma} \mathrm{e}^{\mathrm{i}(\boldsymbol{k}-\boldsymbol{p})\cdot\boldsymbol{r}_0} \Psi_{\boldsymbol{k},\sigma}^{\dagger} \Psi_{\boldsymbol{p},\sigma} \quad (16)$$

where I is the impurity strength.

The scattering t-matrix at the K point is

$$t_{\mathrm{K}} = I\left[1 - I\frac{2}{N_s}\sum_{\boldsymbol{k}} G_{\mathrm{K}}(\boldsymbol{k}, \omega)\right]^{-1}. \quad (17)$$

The discussion about the t-matrix at the K′ point is qualitatively the same, so we only look at the t-matrix at the K point. The sum for $\boldsymbol{k} = (0, k)$, which takes account of the band index $n = 0$ only, is replaced with an integral:

$$\frac{2}{N_s}\sum_{\boldsymbol{k}} G_{\mathrm{K}}(\boldsymbol{k}, \omega) = \rho \int \mathrm{d}\varepsilon \, \frac{1}{\omega^2 - \varepsilon^2} \begin{pmatrix} \omega & -\mathrm{i}\varepsilon \\ \mathrm{i}\varepsilon & \omega \end{pmatrix} \simeq -\rho\pi \mathrm{i} \operatorname{sgn}\omega \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (18)$$

Therefore, we obtain

$$t_{\mathrm{K}} = \frac{I}{1 + I\rho\pi \mathrm{i} \operatorname{sgn}\omega} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (19)$$

The scattering matrix t_{K} is a representation by the A and B sublattices. The transformation into the representation where the $\pm\gamma|k|$ branches are diagonal is performed as follows:

$$t_{\mathrm{K}}' = \frac{1}{2} \begin{pmatrix} 1 & -\mathrm{i} \operatorname{sgn} k \\ 1 & \mathrm{i} \operatorname{sgn} k \end{pmatrix} t_{\mathrm{K}} \begin{pmatrix} 1 & 1 \\ \mathrm{i} \operatorname{sgn} k & -\mathrm{i} \operatorname{sgn} k \end{pmatrix}. \quad (20)$$

The form of t'_{K} becomes

$$t'_{\mathrm{K}} = \frac{I}{1 + I\rho\pi \mathrm{i}\,\mathrm{sgn}\,\omega} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{21}$$

The scattering matrix t'_{K} is diagonal, and the off-diagonal matrix elements vanish. This means that only the scattering processes from k to k and from $-k$ to $-k$ are effective. The scatterings from k to $-k$ and from $-k$ to k are cancelled. Such an absence of backward scattering has been discussed recently [4]. In [4] the authors used the rotation properties of wave functions, and calculated the t-matrix using the explicit forms of the wave functions. Here, we have made our formulation with the scattering t-matrix using the propagators, and have shown that the off-diagonal matrix elements become zero.

4. Impurity scattering with a superconductivity pair potential

In this section, we consider the single-impurity scattering when the superconducting pair potential is present. We look at how the absence of backward scattering discussed in the previous section changes.

In the Nambu representation, the scattering t-matrix at the K point is

$$\tilde{t}_{\mathrm{K}} = \tilde{I}\left[1 - \frac{2}{N_s}\sum_{\boldsymbol{k}} \tilde{G}_{\mathrm{K}}(\boldsymbol{k},\omega)\tilde{I}\right]^{-1} \tag{22}$$

where

$$\tilde{I} = I \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{23}$$

The sign of the scattering potential for holes is reversed from that for electrons, so the minus sign appears in the third and fourth diagonal matrix elements.

The sum over $\boldsymbol{k}$ is performed as in the previous section, and we obtain

$$\frac{2}{N_s}\sum_{\boldsymbol{k}} \tilde{G}_{\mathrm{K}}(\boldsymbol{k},\omega) = \rho \int \mathrm{d}\varepsilon \begin{pmatrix} G^{(1)} & G^{(1,2)} \\ G^{(2,1)} & G^{(2)} \end{pmatrix}. \tag{24}$$

Here, the matrix elements are calculated explicitly, and become as follows:

$$\rho \int \mathrm{d}\varepsilon\, G^{(1)} = \rho \int \mathrm{d}\varepsilon\, G^{(2)} = -\rho\pi \mathrm{i} \frac{\omega}{\sqrt{\omega^2 - \Delta^2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{25}$$

and

$$\rho \int \mathrm{d}\varepsilon\, G^{(1,2)} = \rho \int \mathrm{d}\varepsilon\, G^{(2,1)} = -\rho\pi \mathrm{i} \frac{\Delta}{\sqrt{\omega^2 - \Delta^2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{26}$$

Therefore, we obtain the scattering t-matrix:

$$\tilde{t}_{\mathrm{K}} = \frac{I}{1 + (I\rho\pi)^2} \begin{pmatrix} 1+\alpha\omega & 0 & -\alpha\Delta & 0 \\ 0 & 1+\alpha\omega & 0 & -\alpha\Delta \\ -\alpha\Delta & 0 & -1+\alpha\omega & 0 \\ 0 & -\alpha\Delta & 0 & -1+\alpha\omega \end{pmatrix} \tag{27}$$

where $\alpha = I\rho\pi \mathrm{i}/\sqrt{\omega^2 - \Delta^2}$. In order to look at the dependences on the scattering channels, we should transform to the representation where the wavenumber states are diagonal:

$$\tilde{t}'_{\mathrm{K}} = \frac{1}{2} \begin{pmatrix} 1 & -\mathrm{i}\,\mathrm{sgn}\,k & 0 & 0 \\ 1 & \mathrm{i}\,\mathrm{sgn}\,k & 0 & 0 \\ 0 & 0 & 1 & -\mathrm{i}\,\mathrm{sgn}\,k \\ 0 & 0 & 1 & \mathrm{i}\,\mathrm{sgn}\,k \end{pmatrix} t_{\mathrm{K}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ \mathrm{i}\,\mathrm{sgn}\,k & -\mathrm{i}\,\mathrm{sgn}\,k & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & \mathrm{i}\,\mathrm{sgn}\,k & -\mathrm{i}\,\mathrm{sgn}\,k \end{pmatrix}. \tag{28}$$

The final form is as follows:

$$\tilde{t}'_{\mathrm{K}} = \frac{I}{1+(I\rho\pi)^2}\begin{pmatrix} 1+\alpha\omega & 0 & -\alpha\Delta & 0 \\ 0 & 1+\alpha\omega & 0 & -\alpha\Delta \\ -\alpha\Delta & 0 & -1+\alpha\omega & 0 \\ 0 & -\alpha\Delta & 0 & -1+\alpha\omega \end{pmatrix}. \tag{29}$$

Hence, we find that the off-diagonal matrix elements become zero in the diagonal 2×2 submatrix. This implies that the backward scatterings of electron-like and hole-like quasiparticles vanish in the presence of proximity effects, too. The off-diagonal 2×2 submatrix has diagonal matrix elements whose magnitudes are proportional to Δ. The finite correlation gives rise to backward scatterings of the hole of wavenumber $-k$ when the electron with wavenumber k is incident. The backscatterings of the electrons with wavenumber $-k$ occur for the incident holes with wavenumber k, too. Negative and positive currents induced by two such scattering processes cancel each other. Therefore, the nonmagnetic impurity *does not hinder the supercurrent* in the regions where the superconducting proximity effects occur. This effect is interesting in view of the recent experimental progress as regards superconducting proximity effects [1, 2].

5. Complex pairing

In the previous sections, we have considered the simple *s-wave-like* pairing correlations. In this section, we discuss effects of more complex pairings. In particular, we consider possible pairings between electrons with the dispersion $E = \gamma|k|$ and electrons with the dispersion $E = -\gamma|k|$. Such pairing correlations with the strength Γ will be smaller than the simple correlation with strength Δ. However, the effects of Γ will be interesting, because the above electronic states are degenerate at the Fermi energy.

The pair potential in the representation where the wavenumber states are diagonal has the following form:

$$\begin{aligned} H_{\text{pair}} &= \Delta \sum_{\boldsymbol{k}} (\phi^{(1)\dagger}_{\boldsymbol{k},\uparrow}\phi^{(1)\dagger}_{-\boldsymbol{k},\downarrow} + \phi^{(2)\dagger}_{\boldsymbol{k},\uparrow}\phi^{(2)\dagger}_{-\boldsymbol{k},\downarrow} + \phi^{(3)\dagger}_{\boldsymbol{k},\uparrow}\phi^{(3)\dagger}_{-\boldsymbol{k},\downarrow} + \phi^{(4)\dagger}_{\boldsymbol{k},\uparrow}\phi^{(4)\dagger}_{-\boldsymbol{k},\downarrow} + \text{h.c.}) \\ &\quad + \Gamma \sum_{\boldsymbol{k}} (\phi^{(1)\dagger}_{\boldsymbol{k},\uparrow}\phi^{(2)\dagger}_{-\boldsymbol{k},\downarrow} + \phi^{(2)\dagger}_{\boldsymbol{k},\uparrow}\phi^{(1)\dagger}_{-\boldsymbol{k},\downarrow} + \phi^{(3)\dagger}_{\boldsymbol{k},\uparrow}\phi^{(4)\dagger}_{-\boldsymbol{k},\downarrow} + \phi^{(4)\dagger}_{\boldsymbol{k},\uparrow}\phi^{(3)\dagger}_{-\boldsymbol{k},\downarrow} + \text{h.c.}). \end{aligned} \tag{30}$$

After transforming into the A- and B-sublattice representation, we obtain

$$\begin{aligned} H_{\text{pair}} &= (\Delta + \Gamma) \sum_{\boldsymbol{k}} (\psi^{(1)\dagger}_{\boldsymbol{k},\uparrow}\psi^{(1)\dagger}_{-\boldsymbol{k},\downarrow} + \psi^{(3)\dagger}_{\boldsymbol{k},\uparrow}\psi^{(3)\dagger}_{-\boldsymbol{k},\downarrow} + \text{h.c.}) \\ &\quad + (\Delta - \Gamma) \sum_{\boldsymbol{k}} (\psi^{(2)\dagger}_{\boldsymbol{k},\uparrow}\psi^{(2)\dagger}_{-\boldsymbol{k},\downarrow} + \psi^{(4)\dagger}_{\boldsymbol{k},\uparrow}\psi^{(4)\dagger}_{-\boldsymbol{k},\downarrow} + \text{h.c.}). \end{aligned} \tag{31}$$

The dispersion of the quasiparticles is $E = \pm\Gamma \pm \sqrt{(\gamma k)^2 + \Delta^2}$. The term Γ removes the twofold degeneracy of the dispersion that appeared in the previous section.

The inverse propagator around the K point with $\boldsymbol{k} = (0, k)$ is

$$\tilde{G}_{\mathrm{K}}^{-1}(\boldsymbol{k}, \mathrm{i}\omega_n) = \begin{pmatrix} \mathrm{i}\omega_n & \mathrm{i}\gamma k & -\Delta - \Gamma & 0 \\ -\mathrm{i}\gamma k & \mathrm{i}\omega_n & 0 & -\Delta + \Gamma \\ -\Delta - \Gamma & 0 & \mathrm{i}\omega_n & -\mathrm{i}\gamma k \\ 0 & -\Delta + \Gamma & \mathrm{i}\gamma k & \mathrm{i}\omega_n \end{pmatrix}. \tag{32}$$

The inverse matrix, where $\mathrm{i}\omega_n$ is replaced with ω, is calculated as

$$\tilde{G}_{\mathrm{K}}(k, \omega) = \begin{pmatrix} G^{(1)} & G^{(1,2)} \\ G^{(2,1)} & G^{(2)} \end{pmatrix}. \tag{33}$$

Here, each component is a 2×2 matrix. The explicit forms given using the definition

$$\Omega \equiv [(\omega+\Gamma)^2 - (\gamma k)^2 - \Delta^2][(\omega-\Gamma)^2 - (\gamma k)^2 - \Delta^2]$$

are shown below:

$$G^{(1)} = \frac{1}{\Omega}\begin{pmatrix} \omega[\omega^2 - (\gamma k)^2 - (\Delta-\Gamma)^2] & -\mathrm{i}\gamma k[\omega^2 - (\gamma k)^2 - (\Delta^2-\Gamma^2)] \\ \mathrm{i}\gamma k[\omega^2 - (\gamma k)^2 - (\Delta^2-\Gamma^2)] & \omega[\omega^2 - (\gamma k)^2 - (\Delta+\Gamma)^2] \end{pmatrix} \tag{34}$$

$$G^{(2)} = \frac{1}{\Omega}\begin{pmatrix} \omega[\omega^2 - (\gamma k)^2 - (\Delta-\Gamma)^2] & \mathrm{i}\gamma k[\omega^2 - (\gamma k)^2 - (\Delta^2-\Gamma^2)] \\ -\mathrm{i}\gamma k[\omega^2 - (\gamma k)^2 - (\Delta^2-\Gamma^2)] & \omega[\omega^2 - (\gamma k)^2 - (\Delta+\Gamma)^2] \end{pmatrix} \tag{35}$$

$$G^{(1,2)} = \frac{1}{\Omega}\begin{pmatrix} [\omega^2 - (\mathcal{A})^2](\mathcal{B}) - (\gamma k)^2(\mathcal{A}) & 2\mathrm{i}\omega(\gamma k)\Gamma \\ 2\mathrm{i}\omega(\gamma k)\Gamma & [\omega^2 - (\mathcal{B})^2](\mathcal{A}) - (\gamma k)^2(\mathcal{B}) \end{pmatrix} \tag{36}$$

and

$$G^{(2,1)} = \frac{1}{\Omega}\begin{pmatrix} [\omega^2 - (\mathcal{A})^2](\mathcal{B}) - (\gamma k)^2(\mathcal{A}) & -2\mathrm{i}\omega(\gamma k)\Gamma \\ -2\mathrm{i}\omega(\gamma k)\Gamma & [\omega^2 - (\mathcal{B})^2](\mathcal{A}) - (\gamma k)^2(\mathcal{B}) \end{pmatrix} \tag{37}$$

where

$$\mathcal{A} = \Delta - \Gamma \qquad \mathcal{B} = \Delta + \Gamma.$$

The sum over the wavenumber k is replaced with an integration, as has been done in the previous section. Thus, we obtain

$$\rho \int \mathrm{d}\varepsilon\, G^{(1)} = \rho \int \mathrm{d}\varepsilon\, G^{(2)}$$
$$= -\frac{\rho\pi\mathrm{i}}{2}\begin{pmatrix} \dfrac{\omega+\Delta-\Gamma}{\sqrt{(\mathcal{C})^2-\Delta^2}} + \dfrac{\omega-\Delta+\Gamma}{\sqrt{(\mathcal{D})^2-\Delta^2}} & 0 \\ 0 & \dfrac{\omega-\Delta-\Gamma}{\sqrt{(\mathcal{C})^2-\Delta^2}} + \dfrac{\omega+\Delta+\Gamma}{\sqrt{(\mathcal{D})^2-\Delta^2}} \end{pmatrix} \tag{38}$$

and

$$\rho \int \mathrm{d}\varepsilon\, G^{(1,2)} = \rho \int \mathrm{d}\varepsilon\, G^{(2,1)}$$
$$= -\frac{\rho\pi\mathrm{i}}{2}\begin{pmatrix} \dfrac{\omega+\Delta-\Gamma}{\sqrt{(\mathcal{C})^2-\Delta^2}} - \dfrac{\omega-\Delta+\Gamma}{\sqrt{(\mathcal{D})^2-\Delta^2}} & 0 \\ 0 & -\dfrac{\omega-\Delta-\Gamma}{\sqrt{(\mathcal{C})^2-\Delta^2}} + \dfrac{\omega+\Delta+\Gamma}{\sqrt{(\mathcal{D})^2-\Delta^2}} \end{pmatrix} \tag{39}$$

where

$$\mathcal{C} = \omega - \Gamma \qquad \mathcal{D} = \omega + \Gamma.$$

We can easily check that in the limit $\Gamma \to 0$ these reduce to equations (25) and (26) of the previous section.

Hence, we find that all the 2×2 submatrices of $\tilde{G}_{\mathrm{K}}$ are diagonal, and the off-diagonal matrix elements vanish. This indicates that a scattering t-matrix like equation (29) has the same form, where all the off-diagonal matrix elements of the submatrices become zero. Therefore, the additional pairing potential Γ does not alter the conclusion that the nonmagnetic impurity *does not hinder the supercurrent* in the regions where the superconducting proximity effects are present.

6. Summary

In summary, we have investigated the effects of the superconducting pair potential on the impurity scattering processes in metallic carbon nanotubes. We have used the continuum $\boldsymbol{k} \cdot \boldsymbol{p}$ model for the electronic states, and have considered impurity scattering processes in the normal state and also in the state with the superconducting pair potential. The backward scattering of electrons vanishes in the normal state. In the presence of the superconducting pair correlations, the backward scatterings of electron- and hole-like quasiparticles vanish, too. The impurity gives rise to backward scatterings of holes for incident electrons, and it also induces backward scatterings of electrons for incident holes. Negative and positive currents induced by such scatterings between electrons and holes cancel each other. Therefore, the nonmagnetic impurity does not hinder the supercurrent in the regions where the superconducting proximity effects occur.

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Note added in proof. A short communication of this full paper has appeared as [8].

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